

New bounds on neutrino magnetic moment and re-examination of plasma effect in neutrino spin light

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Summary. — Recent discussion on the possibility to obtain more stringent bounds on neutrino magnetic moment has stimulated new interest to possible effects induced by neutrino magnetic moment. In particular, in this note after a short review on neutrino magnetic moment we re-examine the effect of plasmon mass on neutrino spin light radiation in dense matter. We track the entry of the plasmon mass quantity in process characteristics and found out that the most substantial role it plays is the formation of the process threshold. It is shown that far from this point the plasmon mass can be omitted in all the corresponding physical quantities and one can rely on the results of massless photon spin light radiation theory in matter.

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1. – Neutrino magnetic moment

Neutrino magnetic moments are no doubt among the most well theoretically understood and experimentally studied neutrino electromagnetic properties. [1, 2]

As it was shown long ago [3], in a wide set of theoretical frameworks neutrino magnetic moment is proportional to the neutrino mass and in general very small. For instance, for the minimally extended Standard Model the Dirac neutrino magnetic moment is given by [3]:

$$(1) \quad \mu_{ii} = \frac{3eG_F}{8\sqrt{2}\pi^2} m_i \approx 3.2 \cdot 10^{-19} \left(\frac{m_i}{1\text{eV}} \right) \mu_B.$$

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At the same time, the magnetic moment of hypothetical heavy neutrino (with mass $m_e \ll m_W \ll m_\nu$) is $\mu_\nu = \frac{eG_F m_\nu}{8\sqrt{2}\pi^2}$ [4]. It should be noted here that much larger values for the neutrino magnetic moments are possible in various extensions of the Standard Model (see, for instance, in [1])

Constraints on the neutrino magnetic moment can be obtained in $\nu - e$ scattering experiments from the observed lack of distortions of the recoil electron energy spectra. Recent reactor experiments provides us with the following upper bounds on the neutrino magnetic moment: $\mu_\nu \leq 9.0 \times 10^{-11} \mu_B$ (MUNU collaboration [5]), $\mu_\nu \leq 7.4 \times 10^{-11} \mu_B$ (TEXONO collaboration [6]). The GEMMA collaboration has obtain the world best limit $\mu_\nu \leq 3.2 \times 10^{-11} \mu_B$ [7]. Another kind of neutrino experiment Borexino (solar neutrino scattering) has obtained rather strong bound: $\mu_\nu \leq 5.4 \times 10^{-11} \mu_B$ [8]. The best astrophysical constraint on the neutrino magnetic moment has been obtained from observation of the red giants cooling $\mu_\nu \leq 3 \times 10^{-12} \mu_B$ [9].

As it was pointed out above the most stringent terrestrial constraints on a neutrino effective magnetic moments have been obtained in (anti)neutrino-electron scattering experiments and the work to attain further improvements of the limits is in process. In particular, it is expected that the new bound on the level of $\mu_\nu \sim 1.5 \times 10^{-11} \mu_B$ can be reached by the GEMMA Collaboration in a new series of measurements at the Kalinin Nuclear Power Plant with much closer displacements of the detector to the reactor that can significantly enhanced the neutrino flux(see [7]).

An attempt to reasonably improve the experimental bound on a neutrino magnetic moment was undertaken in [10] where it was claimed that the account for the electron binding effect in atom can significantly increase the electromagnetic contribution to the differential cross section in respect to the case when the free electron approximation is used in calculations of the cross section.

However, as it was shown in a series of papers [11-13] the neutrino reactor experiments on measurements of neutrino magnetic moment are not sensitive to the electron binding effect, so that the free electron approximation can be used for them.

2. – Magnetic moment and neutrino propagation in matter

One may expect that neutrino electromagnetic properties can be much easier visualized when neutrino is propagating in external magnetic fields and dense matter. Also, neutrino propagation in matter is a rather longstanding research field nevertheless still having advances and obtaining a lot of interesting predictions for various phenomena.

The convenient and elegant way for description of neutrino interaction processes in matter has been recently offered in a series of papers [14,15]. The developed method is based on the use of solutions of the modified Dirac equation for neutrino in matter in Feynman diagrams. The method was developed before for studies of different processes in quantum electrodynamics and was called as "the method of exact solutions" [16] The gain from the introduction of the method was sustained by prediction and detailed quantum description of the new phenomenon of the spin light of neutrino in matter (the $SL\nu$), first predicted in [17] within the quasi-classical treatment of neutrino spin evolution. The essence of the $SL\nu$ is the electromagnetic radiation in neutrino transition between two different helicity states in matter.

The simplification of the process framework, such as use of the uniform, unpolarized and non-moving matter, neglect of the matter influence on the radiated photon, makes the estimate of real process relevance in astrophysical settings far from the practical scope. In this short paper we should like to make a step towards the completeness of

the physical picture and to consider the incomprehensible at first glance question of the plasmon mass influence on the $SL\nu$. The importance of plasma effects for the $SL\nu$ in matter was first pointed out in [14]. The investigations already carried out in this area [18] indicated that the plasmon emitted in the $SL\nu$ has a considerable mass that can affect the physics of the process. However the calculation method used there does not lead to the direct confrontation of the results [18] with analogous for the $SL\nu$ [14].

To see how the plasmon mass enters the $SL\nu$ quantities we appeal to the method of exact solutions and carry out all the computations relevant to the $SL\nu$. In this respect, in order to have the conformity we also set all the conditions for the task the same as for corresponding studies on the $SL\nu$. In particular, we consider only the Standard Model neutrino interactions and take matter composed of electrons.

In the exact solutions method, one starts with the modified Dirac equation for the neutrino in matter in order to have initial and final neutrino states, which would enter the process amplitude. The equation reads as follows [14]:

$$(2) \quad \{i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu(1 + \gamma^5)f^\mu - m\}\Psi(x) = 0,$$

where in the case of neutrino motion through the non-moving and unpolarized matter $f^\mu = G_f/\sqrt{2} (n, \mathbf{0})$ with n being matter (electrons) number density. Under this conditions the equation (2) has plane-wave solution determined by 4-momentum p and quantum numbers of helicity $s = \pm 1$ and sign of energy $\varepsilon = \pm 1$. For the details of equation solving and exact form of the wave functions $\Psi_{\varepsilon,p,s}(\mathbf{r}, t)$ the reader is referred to [14] and [15], here we cite only the expression for the neutrino energy spectrum:

$$(3) \quad E = \varepsilon \sqrt{(p - s\tilde{n})^2 + m_\nu^2} + \tilde{n}, \quad \tilde{n} = \frac{1}{2\sqrt{2}}G_F n.$$

The S-matrix of the process involves the usual dipole electromagnetic vertex $\mathbf{\Gamma} = i\omega\{[\mathbf{\Sigma} \times \mathbf{\kappa}] + i\gamma^5\mathbf{\Sigma}\}$ and for given spinors for the initial and final neutrino states $u_{i,f}$ can be written as

$$(4) \quad S_{fi} = -(2\pi)^4 \mu \sqrt{\frac{\pi}{2\omega L^3}} \delta(E_2 - E_1 + \omega) \delta^3(\mathbf{p}_2 - \mathbf{p}_1 + \mathbf{k}) \bar{u}_f(\mathbf{e}, \mathbf{\Gamma}_{fi}) u_i.$$

Here \mathbf{e} is the photon polarization vector, μ is the transitional magnetic moment and L is the normalization length. The delta-functions before spinors convolution part lead to the conservation laws

$$(5) \quad E_1 = E_2 + \omega; \quad \mathbf{p}_1 = \mathbf{p}_2 + \mathbf{k},$$

with energies for the initial and final neutrinos $E_{1,2}$ taken in accordance to (3). For the photon dispersion, for the purpose of our study it is sufficient to use the simplest expression

$$(6) \quad \omega = \sqrt{k^2 + m_\gamma^2}.$$

As it was discussed in our previous studies on the $SL\nu$ [14, 15] the most appropriate conditions for the radiation to manifest its properties are met in dense astrophysical

objects. This is the setting we will use further for the process and in the case of cold plasma the plasmon mass should be taken as

$$(7) \quad m_\gamma = \sqrt{2\alpha}(3\sqrt{\pi n})^{1/3}.$$

The numerical evaluation at typical density gives $m_\gamma \sim 10^8 eV$, while the density parameter $\tilde{n} \sim 10^4 eV$.

3. – Plasmon mass influence

Let us now consider the influence of dense plasma on the process of spin light of neutrino. Similarly to the original spin light calculation we consider the case of initial neutrino possessing the helicity quantum number $s_1 = -1$ and the corresponding final neutrino helicity is $s_2 = 1$. Using the neutrino energies (3) with corresponding helicities one can resolve the equations (5) in relation to plasmon momentum which is not equal to its energy since we take into account the dispersion of the emitted photon in plasma (6).

For convenience of calculations it is possible to use the following simplification. In most cases the neutrino mass appeared to be the smallest parameter in the considered problem and it is several orders smaller than any other parameter in the system. So we could first examine our process in approximation of zero neutrino mass, though we should not forget that only neutrino with non-zero mass could naturally possess the magnetic moment. This our simplification should be considered only as a technical one. It should be pointed here that in order to obtain the consistent description of the $SL\nu$ one should account for the effects of the neutrino mass in the dispersion relation and the neutrino wave functions.

From the energy-momentum conservation it follows [18] that the process is kinematically possible only under the condition (taking account of the above-mentioned simplification):

$$(8) \quad \tilde{n}p > \frac{m_\gamma^2}{4}.$$

Provided with the plasmon momentum we proceed with calculation of the $SL\nu$ radiation rate and total power. The exact calculation of total rate is an intricate problem and the final expression is too large to be presented here. However one can consider the most notable ranges of parameters to investigate some peculiarities of the rate behavior.

First of all we calculate the rate for the case of the $SL\nu$ without plasma influence. This can be done by choosing the limit $m_\gamma \rightarrow 0$ and the obtained result is in full agreement with [14]:

$$(9) \quad \Gamma = 4\mu^2\tilde{n}^2(\tilde{n} + p).$$

From (9) one easily derives the $SL\nu$ rate for two important cases, *i.e.* high and ultra-high densities of matter just by choosing correspondingly p or \tilde{n} as the leading parameter in the brackets. While neutrino mass is the smallest quantity, our system fall within the range of relativistic initial neutrino energies.

The corresponding expression for the total power also covers high and ultra-high density cases [14] as well as the intermediate area where the density parameter and the neutrino momentum are comparable:

$$(10) \quad I = \frac{4}{3}\mu^2\tilde{n}^2(3\tilde{n}^2 + 4p\tilde{n} + p^2).$$

If we account for the plasma influence (thus, $m_\gamma \neq 0$) on the $SL\nu$ we can discuss two important situations. One is the area of parameters near the threshold, and the other is connected with direct contribution of m_γ into the radiation rate expression. The later case is particularly important for this study, because it fulfill the aim of the present research in finding the conditions under which the plasmon mass can not be neglected.

For physically reliable conditions the density parameter usually appears to be less then the plasmon mass, which in its turn is less then the neutrino momentum: $\tilde{n} \ll m_\gamma \ll p$. Obviously the threshold condition (8) should be satisfied. As we consider the conditions similar to different astrophysical objects it is natural to use high-energy neutrino.

Using the series expansion of the total rate one could obtain the rate of the process in the following form:

$$(11) \quad \Gamma = 4\mu^2 p \tilde{n}^2 (1 + 6\lambda + 4\lambda \ln \lambda),$$

where $\lambda = \frac{m_\gamma^2}{4\tilde{n}p} < 1$. Approaching the threshold ($\lambda \rightarrow 1$), the expansion (11) becomes inapplicable, however it is correct in rather wide range of parameters with $m_\gamma \ll p$ and $\tilde{n} \ll p$. Near the threshold the the total rate can be presented in the form $\Gamma \sim (1 - \lambda)$ but the exact coefficient is too unwieldy to be presented here.

Concerning the power of the $SL\nu$ with plasmon, one can use the expansion:

$$(12) \quad I = \frac{4}{3}\mu^2 p^2 \tilde{n}^2 (1 - 6\lambda - 57\lambda \frac{\tilde{n}}{p} - 12\lambda \frac{\tilde{n}}{p} \ln \lambda).$$

The expression (12) is correct only if the system meets the requirement $\lambda \ll 1$. Otherwise one should use higher orders of quantity $\frac{m_\gamma^2}{p}$ in the expansion to achieve a reliable value of intensity. Near the threshold the power has the same dependence on the "distance" from the threshold $(1 - \lambda)$ as the rate of the process.

4. – Conclusion

There is an increasing interest to neutrino electromagnetic properties and neutrino magnetic moments in particular. This interest is stimulated, first by the progress in experimental bounds on magnetic moments which have been recently achieved, as well as theoretical predictions of new processes emerging due to neutrino magnetic moment, such as the $SL\nu$ and a believe in its importance for possible astrophysical applications.

Further developing the theory of the spin light of neutrino, we have explicitly shown that the influence of plasmon mass becomes significant (see (11) and (12)) when the parameter λ is comparable with 1, this corresponds to the system near the threshold. As soon as the quantity $\lambda \ll 1$ (so the system is far from the threshold) one can use either $SL\nu$ radiation rate and total power from [14] or their rather compact generalizations (11) and (12) where the plasmon mass is accounted for as a minor adjustment.

Since high energy neutrinos propagating in matter could be rather typical situation in astrophysics, for instance in neutron stars, the influence of photon dispersion in plasma on the $SL\nu$ process can be neglected and the threshold generated by the non-zero plasmon mass should not be taken into account. However, the method of exact solutions of Modified Dirac equation provides us with analytical expressions for probability and intensity in the whole range of possible parameters.

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